

## Mechanism for Designing Metallic Metamaterials with a High Index of Refraction

J. T. Shen, Peter B. Catrysse, and Shanhui Fan\*

Ginzton Laboratory, Stanford University, Stanford, California 94305, USA  
(Received 8 November 2004; published 17 May 2005)

We introduce a mechanism for creating artificial high refractive index metamaterials by exploiting the existence of subwavelength propagating modes in metallic systems. As an example, we investigate analytically and numerically metal films with a periodic arrangement of cut-through slits. Because of the presence of TEM modes in the slits, for TM polarization such a system can be rigorously mapped into a high refractive index dielectric slab when the features are smaller than the wavelength of light. The effective refractive index is entirely controlled by the geometry of the metal films, is positive, frequency independent, and can be made arbitrarily large.

DOI: 10.1103/PhysRevLett.94.197401

PACS numbers: 78.20.Ci, 41.20.Jb, 42.25.Bs, 42.79.Dj

Recently, there has been great interest in exploiting subwavelength resonances in metallic structures to create artificial materials with unusual effective electromagnetic responses. Notable examples include high-impedance surfaces used as an antenna substrate [1–3], negative refractive index metamaterials [4–7], effective surface plasmon behavior [8] on perfect metal surface with gratings [9], and effective bulk plasmon behavior [10] in thin-wire structures [11–16]. In this Letter, we show that a perfect metal film with a periodic arrangement of cut-through slits can be regarded as a dielectric slab with a frequency-independent effective refractive index. The effective index in this system is entirely controlled by geometry, and indices that are arbitrarily high can be straightforwardly synthesized. Such a capability is potentially important for miniaturization of optical or electromagnetic devices and for improving resolution in imaging. More fundamentally, the refractive index is commonly regarded as an intrinsic material property that is directly related to the underlying electronic states. By pointing out that the refractive index can be controlled by geometry only, and that ranges of large refractive index that are not previously accessible can in fact be generated with metamaterials, this work adds evidence to the important potential of replacing electronic states with subwavelength electromagnetic resonances, which could open up a new world of possibilities in optical physics.

The key to creating the desired effective index behavior lies in the existence of subwavelength propagating modes in metallic structures. As an example, let us consider a metal film with one-dimensional periodic cut-through slits, as shown in Fig. 1(a). Without losing generality, we assume hereafter vacuum for the ambient environment and the slit regions. In the slits, regardless of how small the width is, there exists a propagating TEM mode, with the electric field pointing in the  $x$  direction. It has been shown [17–20] that the presence of this mode permits perfect transmission of light through subwavelength slit arrays. Here, in order to define an appropriate effective index, we focus on the behavior of this structure in the long wavelength limit. As it turns out, the properties of the metal film for the TM polarization asymptotically approach those of a di-

electric slab with a uniquely defined refractive index  $n$  and a width  $\bar{L}$ , as depicted in Fig. 1(b).

We first give a heuristic derivation to establish this connection. Our heuristics are based on fundamental concepts such as resonance, instantaneous power flow, and energy conservation. Specifically, the Fabry-Perot resonance condition for the TEM mode in the slit is  $k \times L \approx m\pi$ , where  $k$  is the wave vector of the incident light,  $L$  is the thickness of the metal film, and  $m$  is a positive integer. This condition can be written as  $(nk) \times (L/n) \approx m\pi$  as well for any numerical value of  $n$ , which implies that any dielectric slab with refractive index  $n$  and thickness  $\bar{L} \equiv L/n$  would give the same Fabry-Perot resonances. To uniquely determine  $n$ , we compare the fields in both systems. We will use an overhead line over a variable to denote the corresponding quantity in the effective medium hereafter. Suppose that the effective homogeneous electric field in the effective medium is  $\overline{\mathbf{E}}$ ; we then have  $\overline{\mathbf{E}} = \frac{a}{d}\mathbf{E}$ . Requiring the instantaneous energy flow across the surface

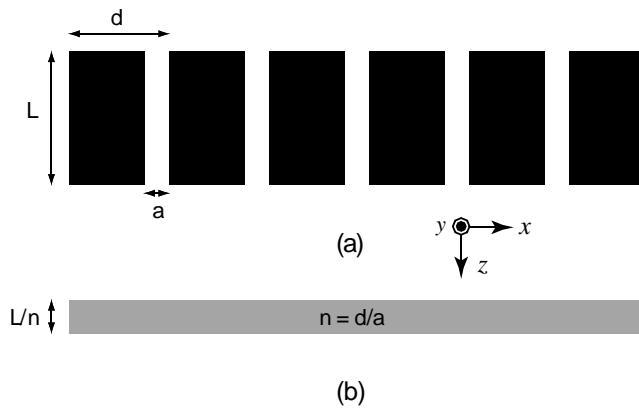


FIG. 1. (a) Schematic of the metal film with periodic slits. The parameters are defined as in the figure:  $a$  is the width of the slit,  $d$  is the periodicity, and  $L$  is the thickness of the metal film. The black regions indicate the metal parts, and the white regions are the vacuum. The film is extended in the  $x - y$  plane. (b) The equivalent effective dielectric slab corresponding to (a). The effective refractive index is  $n \equiv d/a$ , and the thickness is  $\bar{L} \equiv L/n$ .

<b>Report Documentation Page</b>			Form Approved OMB No. 0704-0188		
<p>Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.</p>					
1. REPORT DATE <b>NOV 2004</b>	2. REPORT TYPE	3. DATES COVERED <b>00-00-2004 to 00-00-2004</b>			
4. TITLE AND SUBTITLE <b>Mechanism for Designing Metallic Metamaterials with a High Index of Refraction</b>			5a. CONTRACT NUMBER		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S)			5d. PROJECT NUMBER		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Stanford University, Ginzton Laboratory, Stanford, CA, 94305</b>			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)			10. SPONSOR/MONITOR'S ACRONYM(S)		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S)		
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:  a. REPORT      b. ABSTRACT      c. THIS PAGE <b>unclassified</b> <b>unclassified</b> <b>unclassified</b>			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>	18. NUMBER OF PAGES <b>4</b>	19a. NAME OF RESPONSIBLE PERSON

to be the same for both the metal film and the effective dielectric slab,  $(\mathbf{E} \times \mathbf{H})_z \times a = (\overline{\mathbf{E}} \times \overline{\mathbf{H}})_z \times d$ , we obtain the scaling condition for the magnetic fields  $\overline{\mathbf{H}} = \mathbf{H}$ . Note that, for the TEM mode considered here, the only nonzero fields are  $E_x$  and  $H_y$ . In addition, Maxwell's equations give the following relations between the electric and magnetic fields:  $\overline{\mathbf{E}}/\overline{\mathbf{H}} = \sqrt{\mu/\epsilon}$  and  $\mathbf{E}/\mathbf{H} = \sqrt{\mu/\epsilon}$ . By imposing the condition that the total energy for both systems is the same for any time  $t$ ,

$$\frac{1}{2}(\overline{\epsilon}\overline{\mathbf{E}}^2 + \overline{\mu}\overline{\mathbf{H}}^2) \times \overline{L} \times d = \frac{1}{2}(\epsilon\mathbf{E}^2 + \mu\mathbf{H}^2) \times L \times a, \quad (1)$$

we have  $\overline{\mu} = \mu$  and  $\overline{\epsilon}/\epsilon = (d/a)^2$ , and therefore the effective refractive index is  $n = d/a$ . Consequently, an effective refractive index can be synthesized entirely by geometrical means. This allows the creation of arbitrary frequency-independent refractive indices larger than 1 without having to change the intrinsic electronic states of the material.

We now confirm this intuitive argument by performing rigorous calculations. Consider a TM wave (the magnetic fields pointing in the  $y$  direction) incident on the metal film. The transmission amplitude for the  $p$ th diffraction order is given by [18–20]

$$t_p = \frac{\omega/c}{\alpha_p} g_p \left( \frac{4fu}{(1+\phi)^2 - (1-\phi)^2 u^2} \right), \quad (2)$$

where  $\omega$  is the frequency,  $c$  is the speed of light,  $G_p = k_x + 2\pi p/d$  is the parallel quasimomentum along the metal surface of the  $p$ th diffraction order,  $\alpha_p \equiv \sqrt{(\omega/c)^2 - G_p^2}$  is the momentum in the  $z$  direction,  $f \equiv a/d$  is the area filling factor of the slits,  $u \equiv e^{i(\omega/c)L}$  is the phase accumulation across the metal film with thickness  $L$ ,  $g_p \equiv \text{sinc}(G_p a/2)$ , and

$$\phi = \sum_{p=-\infty}^{\infty} f g_p \frac{\omega/c}{\alpha_p}. \quad (3)$$

At normal incidence,  $g_0 = 1$  and  $\alpha_0 = \omega/c$ . The zeroth order amplitude, which is the amplitude of the only field that propagates afar when the wavelength  $\lambda$  is larger than the periodicity  $d$ , can be written as

$$t_0 = \frac{4[(f/\phi^2)/(1+1/\phi)^2]e^{i(\omega/c)L}}{1 - [(1-1/\phi)/(1+1/\phi)]^2 e^{2i(\omega/c)L}}. \quad (4)$$

Comparing this to the transmission amplitude of a plane wave normally incident on a dielectric slab with refractive index  $n$  and thickness  $\overline{L}$ ,

$$t = \frac{4[n/(1+n)^2]e^{in(\omega/c)\overline{L}}}{1 - [(1-n)/(1+n)]^2 e^{2in(\omega/c)\overline{L}}}, \quad (5)$$

it follows immediately that by identifying  $n = 1/\phi$  and  $\overline{L} = L/n$ , these two amplitudes are asymptotically identical. Note that, in this case,  $1/\phi \approx 1/f = d/a$ , while higher diffraction orders due to the periodicity introduce

a small purely imaginary correction. It is easy to show that the identification becomes very accurate for a large  $d/a$  ratio, i.e., high effective refractive index  $n$ . This confirms the notion of the corresponding “effective dielectric slab” given earlier by the heuristic argument: the transmission properties of a metal film with slits are asymptotically identical to those of a dielectric slab with the refractive index  $n = d/a$  and the thickness  $\overline{L} = L/n$ . This identification to the dielectric slab remains valid for all oblique incident angles as well.

The transmission spectrum, for normally incident light, is shown in Fig. 2(a) for the case  $n = 4$ . This spectrum is calculated using the exact analytic expression Eq. (4). The transmission coefficient becomes 100% when  $k \times L \equiv \omega/c \times L \approx m\pi$ , as expected from the Fabry-Perot resonance condition in the slit. Remarkably, the entire spectrum matches almost perfectly with that of a high index slab with  $n = d/a$  and  $\overline{L} = L/n$ . By increasing  $\overline{L}$  by only 2.5%, the two spectra can be made to completely overlap with each other in the frequency region shown in the figure. For obliquely incident light, the transmission spectrum is shown in Fig. 2(b) as a function of the incident angle. The same correction factor of  $\overline{L}$  from Fig. 2(a) also brings the two spectra in Fig. 2(b) to coincide. Note that the correction factor for  $\overline{L}$  vanishes for larger  $n$ .

With the transmission equivalence established, it would be of great interest if the notion of the effective dielectric slab could be carried over to other optical properties, such as wave-guiding properties along the medium. As noted by the expression for  $\alpha_p$ , i.e., the momentum of the  $p$ th diffraction order field external to the metal film in the  $z$  direction, when the periodicity of the slits is much smaller than the free-space wavelength of light, all diffraction orders are evanescent in the  $z$  direction. These diffraction orders form sizable high intensity bulges centered around the openings of the slits [20]. When these bulges overlap with each other and create propagating modes in the structure, the metal film could act as a waveguide. Recently, it has been shown that structured surfaces on perfect metal can support surface modes that are evanescent inside the structures [9]. In contrast, in our system, due to the prop-

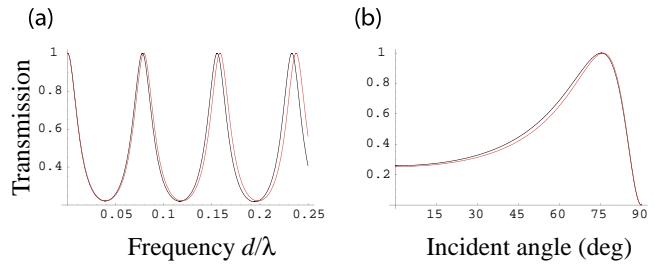


FIG. 2 (color). The transmission coefficient of the metal film (black lines) and the corresponding effective dielectric slab (red lines). The thickness of the metal film is  $L/d = 25/4$ . The effective refractive index of the slab is  $n = d/a = 4$ . (a) Spectrum at normal incidence. (b) Transmission at oblique incidence for the frequency  $\omega = 0.05 \times d/\lambda$ .

agitating nature of the TEM modes in the slits, the guided modes are not surface states but rather closely resemble waveguide modes in a dielectric slab.

The properties of such guided modes can be solved for analytically. When there is no external source present, the magnetic fields in the  $m$ th slit, above the metal film and below the metal film, have the following forms:

$$\begin{aligned} H_y^m(x, z) &= \left[ A^m(x) \cos\left(\frac{\omega}{c}z\right) + B^m(x) \sin\left(\frac{\omega}{c}z\right) \right] \\ \text{for } |x - md| &\leq \frac{a}{2}, \\ H_y^{\text{above}}(x, z) &= e^{ik_x x} \sum_{p=-\infty}^{\infty} r_p e^{i\alpha_p(-z-L/2)} e^{i(2\pi p/d)x}, \\ H_y^{\text{below}}(x, z) &= e^{ik_x x} \sum_{p=-\infty}^{\infty} t_p e^{i\alpha_p(z-L/2)} e^{i(2\pi p/d)x}, \end{aligned} \quad (6)$$

respectively;  $A^m$ ,  $B^m$ ,  $r_p$ , and  $t_p$  are the amplitudes. The electric fields are then obtained from  $\epsilon \partial \mathbf{E} / \partial t = \nabla \times \mathbf{H}$ .

The even eigenmodes ( $r_p = t_p$  for all  $p$ , and  $B^m = 0$  for all  $m$ ) satisfy the dispersion relation

$$\frac{1}{i\phi} = \tan\left(\frac{\omega}{c} \frac{L}{2}\right), \quad (7)$$

and the odd eigenmodes ( $r_p = -t_p$  for all  $p$ , and  $A^m = 0$  for all  $m$ ) satisfy the dispersion relation

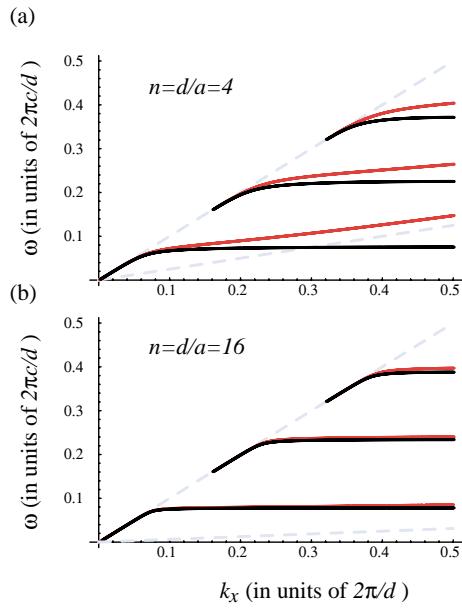


FIG. 3 (color). Dispersion curves of the waveguide modes in the first Brillouin zone. Shown in the figures are the first three even modes of the metal films (black lines) and the corresponding effective dielectric slabs (red lines). The dashed lines are the light lines in vacuum and in the slab waveguides, respectively.  $L/d = 25/4$ . (a)  $n = 4$ ; (b)  $n = 16$ . (The odd modes, not shown here, are located between the even bands and behave similarly.)

$$\frac{1}{i\phi} = -\cot\left(\frac{\omega}{c} \frac{L}{2}\right), \quad (8)$$

as calculated by matching the boundary conditions [19,20]. In both cases,  $\phi = \sum_{p=-\infty}^{\infty} f g_p^2 \frac{\omega/c}{\alpha_p}$ , as defined in Eq. (3), is now purely imaginary for waveguide modes. When we compare Eqs. (7) and (8) with the corresponding dispersion relations of the TM modes of a dielectric slab [21] with refractive index  $n$  and thickness  $L = L/n$ , the mapping  $n = 1/(f g_0^2)$  becomes asymptotically exact for large  $n$ . In practice,  $k_x a \ll 1$ , so  $g_0 = \text{sinc}(k_x a/2) \approx 1$  and therefore  $n \approx 1/f = d/a$ , which is the same condition as for transmission. Figure 3 shows the dispersion curves of the first three even waveguide modes in the first Brillouin zone of the metal films and the corresponding effective dielectric slabs. At low frequency, the wave propagates at the speed of light in the external region, while at large propagating constant  $k_x$ , the eigenfrequencies approach  $(2p-1) \times \pi c/L$  (for even modes) and  $2p \times \pi c/L$  (for odd modes), respectively, where  $p$  is a positive integer. The even and odd modes stack up alternatively and are equally spaced at large  $k_x$ . As expected, the agreement between the metallic system and the effective dielectric slab improves as  $n$  increases. For  $n = 16$ , the two dispersion curves almost completely overlap within the first Brillouin zone. Both the transmission and the waveguide properties of the metal film give the same expression for the effective refractive

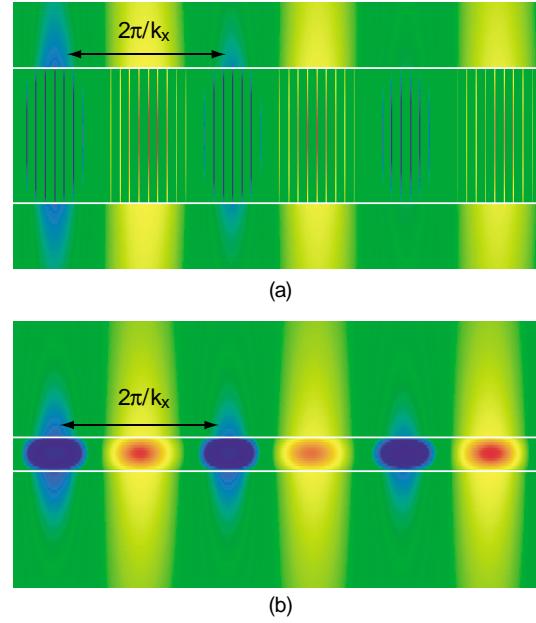


FIG. 4 (color). Snapshots of the  $H_y$  field distributions of the fundamental waveguide modes of the metal film [shown in (a)] and the corresponding effective dielectric slab [shown in (b)] for  $n = d/a = 4$ ,  $L/d = 25/4$ . The red color indicates positive amplitude, while the blue color indicates negative amplitude. The white lines in (a) and (b) outline the film and the slab, respectively. The normalized excitation frequency is  $\omega = d/\lambda = 0.0516$ . The arrows indicate the periodicity of the field.

index  $n = d/a$ , thereby validating the generality of the effective refractive index.

Figure 4 shows the modal patterns of the fundamental waveguide modes of a metal film and the corresponding effective dielectric slab with the refractive index  $n = 4$ . These field distributions are computed using a two-dimensional finite-difference time-domain method. The normalized excitation frequency  $\omega = d/\lambda = 0.0516$  is in the overlap region of the two dispersion curves in Fig. 3(a). The spatial periodicity of the waveguide modes clearly demonstrates the equivalence of the two systems. The fundamental mode in the metallic structure peaks at the center just as the corresponding guided mode in the dielectric slab does, and this distinctly differs from the surface mode behaviors previously considered [9].

For TE polarization (the electric field is pointing in the  $y$  direction), the slits support only evanescent modes. An argument in the same fashion as in Ref. [9] gives  $\bar{\mu}_x/\mu_x = 8/\pi^2 \times a/d$  and a dielectric function of the plasma form:

$$\frac{\bar{\epsilon}_y}{\epsilon_y} = \frac{\pi^2}{8} \frac{d}{a} \left( 1 - \frac{\omega_p^2}{\omega^2} \right). \quad (9)$$

where the geometrical plasma frequency  $\omega_p = \pi c / (a \sqrt{\epsilon_y \mu_x})$ ;  $\epsilon_y$  and  $\mu_x$  are the dielectric constant and permeability, respectively, of any material that may be filling the slits. The limiting wavelength  $\lambda_g$  for this polarization is  $2a$ . Since the structure considered in this Letter is two-dimensional, its behavior is strongly polarization dependent. The mechanism of creating effective high refractive index dielectric structures is not restricted to two dimensions. In three dimensions, subwavelength propagating modes exist in many transmission-line geometries [22], which may be used to create high refractive index materials in three dimensions.

As a final remark, we compare our approach to other attempts to alter the effective refractive index geometrically. The Maxwell Garnett effective medium approximation [23] has been fairly successful in explaining the transmission spectra of the composite metal-insulator and semiconductor-insulator systems [24]; an analytic expression of the effective dielectric constant has also been given for one-dimensional grating systems [25]. Nonetheless, these approaches are not valid when the dielectric constant of the metal becomes large and negative. The results presented here can be directly applied in the wavelength range from microwave to far infrared, where the loss and the plasmonic effects of the metals can be largely neglected. In the optical wavelength range, the presence of the plasmonic response leads to additional subwavelength propagating modes [26–28], which may also be exploited in creating novel optical materials.

J. T. S. thanks P. M. Platzman for drawing his attention to this subject, and M. L. Povinelli for many helpful suggestions on the manuscript. This work is supported in part by NSF Grant No. ECS-0134607 and by AFOSR Grant No. FA9550-04-1-0437.

\*Electronic address: shanhui@stanford.edu

- [1] D. F. Sievenpiper, E. Yablonovitch, J. N. Winn, S. Fan, P. R. Villeneuve, and J. D. Joannopoulos, Phys. Rev. Lett. **80**, 2829 (1998).
- [2] D. Sievenpiper, L. Zhang, R. F. J. Broas, N. G. Alexopolous, and E. Yablonovitch, IEEE Trans. Microwave Theory Tech. **47**, 2059 (1999).
- [3] B. A. Munk, *Frequency Selective Surfaces: Theory and Design* (John Wiley & Sons, New York, 2000).
- [4] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).
- [5] D. R. Smith and N. Kroll, Phys. Rev. Lett. **85**, 2933 (2000).
- [6] S. O'Brien and J. B. Pendry, J. Phys. Condens. Matter **14**, 6383 (2002).
- [7] F. J. Rachford, D. L. Smith, P. F. Loschialpo, and D. W. Forester, Phys. Rev. E **66**, 036613 (2002).
- [8] A. D. Boardman, *Electromagnetic Surface Modes* (John Wiley, New York, 1982).
- [9] J. B. Pendry, L. Martín-Moreno, and F. J. García-Vidal, Science **305**, 847 (2004).
- [10] M. S. Kushwaha, Surf. Sci. Rep. **41**, 1 (2001).
- [11] J. B. Pendry, A. J. Holden, W. J. Stewart, and I. Youngs, Phys. Rev. Lett. **76**, 4773 (1996).
- [12] D. Felbacq and G. Bouchitte, Waves Random Media **7**, 245 (1997).
- [13] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, J. Phys. Condens. Matter **10**, 4785 (1998).
- [14] P. A. Belov, C. R. Simovski, and S. A. Tretyakov, Phys. Rev. E **66**, 036610 (2002).
- [15] R. Marqués, J. Martel, F. Mesa, and F. Medina, Phys. Rev. Lett. **89**, 183901 (2002).
- [16] B. T. Schwarts and R. Piestun, J. Opt. Soc. Am. B **20**, 2448 (2003).
- [17] V. P. Shestopalov *et al.*, *Diffraction Gratings* (Naukova Dumka, Kiev, 1986).
- [18] J. A. Porto, F. J. García-Vidal, and J. B. Pendry, Phys. Rev. Lett. **83**, 2845 (1999).
- [19] P. Lalanne, J. P. Hugonin, S. Astilean, M. Palamaru, and K. D. Möller, J. Opt. A. Pure Appl. Opt. **2**, 48 (2000).
- [20] J. T. Shen and P. M. Platzman, Phys. Rev. B **70**, 035101 (2004).
- [21] P. Yeh, *Optical Waves in Layered Media* (John Wiley & Sons, New York, 1988).
- [22] D. M. Pozar, *Microwave Engineering* (John Wiley & Sons, New York, 1997).
- [23] J. C. Maxwell Garnett, Philos. Trans. R. Soc. Lond. **203**, 385 (1904).
- [24] B. Abeles and J. I. Gittleman, Appl. Opt. **15**, 2328 (1976).
- [25] L. Pilozzi, A. D. Andrea, and R. D. Sole, Phys. Rev. B **54**, 10 751 (1996).
- [26] E. N. Economou, Phys. Rev. **182**, 539 (1969).
- [27] T. W. Ebbesen, H. J. Lezec, H. F. Ghaemi, T. Thio, and P. A. Wolff, Nature (London) **391**, 667 (1998).
- [28] H. Shin, M. F. Yanik, and S. Fan, Appl. Phys. Lett. **84**, 4421 (2004).